

Report

Activities developed in Competition for Young
Researchers - ACRI - Young Investigator Training
Program 2017

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1 Scientific activities developed

The scientific activities regarding the program "Competition for Young Researchers", supported by ACRI - Young Investigator Training Program 2017 and by Università degli Studi di Urbino Carlo Bo, has started during the conference "Two nonlinear days in Urbino 2018", in July 12th to 13th, 2018, organized by the Dipartimento di Scienze Pure e Applicate (DiSPeA) - Università degli Studi di Urbino Carlo Bo, where the researcher Marcos Tadeu de Oliveira Pimenta has presented the following talk,

Title: On existence and concentration of solutions to a class of quasilinear problems involving the 1-Laplace operator.

Abstract: In this work we use variational methods to prove results on existence and concentration of solutions to a problem in \mathbb{R}^N , involving the 1-Laplacian operator. A thorough analysis on the energy functional defined in the space of functions of bounded variation $BV(\mathbb{R}^N)$ is necessary, where the lack of compactness is overcome by using the Concentration-Compactness Principle of Lions.

In a second moment, from October 1st to 31st, 2018, the scientific activities regarding the joint work between the Researcher and the Supervisor took place. Below, more details about this, which consists in the main part of the project, are given.

First of all, it has been studied a non-local version of an obstacle problem, that is the study of the equilibrium position of a string or a membrane constrained to stay under (or above) a prescribed obstacle. There are many other contexts in which variational inequalities appear, such as, just to name a few, fluid filtration in porous media, elastoplasticity, optimal control (see, e.g., the stopping time problem), statistics, economics and financial mathematics (see, e.g. the real options approach to investments).

Many extensions of this problem has been considered in the literature, particularly for taking into account nonlinear elastic reactions of the membrane, non commutative effects and nonlocal interactions. When replacing the local elastic reaction with a nonlocal one, with the purpose of taking into account the long-range interactions of particles, for instance, the standard Laplacian $-\Delta$ might be replaced with the fractional Laplacian

$(-\Delta)^s$, $s \in (0, 1)$, which, up to a positive constant, may be defined as

$$-(-\Delta)^s u(x) = \int_{\mathbb{R}^N} \frac{u(x+y) + u(x-y) - 2u(x)}{|y|^{N+2s}} dy, \quad x \in \mathbb{R}^N, \quad (1.1)$$

In the work developed in the month of October, it has been investigated a nonlocal variational inequality, driven by the fractional Laplacian $(-\Delta)^s$, with the aim to give some existence results of solutions using variational and topological techniques. In particular it has been used some techniques and arguments widely exploited for the investigation of variational inequalities driven by the Laplace operator or more general elliptic operators, in order to show that these methods work also in handling nonlocal problems. More precisely, it has been studied the non-local variational inequality

$$\begin{cases} u \in X_0^s(\Omega), u \leq \psi \text{ a.e. in } \Omega, \\ \langle u, v - u \rangle_{X_0^s(\Omega)} - \lambda \langle u, v - u \rangle_2 \geq \int_{\Omega} f(x, u(x), (-\Delta)^\beta u(x)) (v(x) - u(x)) dx, \\ \text{for any } v \in X_0^s(\Omega), v \leq \psi \text{ a.e. in } \Omega, \end{cases} \quad (1.2)$$

where the following result has been proved:

Theorem 1.1 *Let $s \in (0, 1)$ and $0 < \beta < s/2$. Assume that f verifies $(f_1) - (f_5)$ with $p < 2$ in (f_3) and ψ satisfies (ψ) . Then, for all $0 < \lambda < \lambda_1(s)$, there exists a nontrivial nonnegative solution $u \in C^{0,s}(\overline{\Omega})$ for the variational inequality (1.2).*

In the proof of Theorem 1.1, some difficulties arise from the regularity of the solutions of elliptic problems involving the fractional laplacian operator, what makes difficult to give the pontual estimates necessary to deal with (1.2). Despite these difficulties, in the month of October, 2018, Researcher and Supervisor succeed in proving Theorem 1.1, what gives rise to the following scientific paper, which is now submitted for publication,

Pimenta, M. T. O., Servadei, R. - Some existence results for variational inequalities with nonlocal fractional operators.

2 Work in progress

The activities developed during the program includes also a second result, consisting in the study of a nonlocal version of a variational inequality involving a possible discontinuous reaction term. This study is being carried by Marcos Pimenta, Raffaella Servadei and also

by Giovanni Molica Bisci from Università Mediterranea di Reggio Calabria, which has also a great experience in this field of research.

October 31st, 2018