**PROPOSED RESEARCH PROJECT DURING THE VISITING PERIOD**

Our intention is to study a particular Liouville type problem in a compact surface $\Sigma$ with boundary equipped with a certain metric $g$. Consider the following boundary value problem:

\[
\begin{align*}
-\Delta_g u + 2\bar{K} &= 2Ke^u & \text{in } \Sigma, \\
\frac{\partial u}{\partial n} + 2h &= 2he^{u/2} & \text{on } \partial\Sigma.
\end{align*}
\]

This kind of equations has a special interest due to its geometric and physical meaning. Indeed, the problem allows us to prescribe Gaussian curvature in $\Sigma$ and geodesic curvature on $\partial\Sigma$.

Recently, some progresses have been made in [LS-Malchiodi-Ruiz, 2018] considering the following energy functional $I : H^1(\Sigma) \to \mathbb{R}$,

\[
I(u) = \int_\Sigma \left( \frac{1}{2} |\nabla u|^2 + 2\bar{K}u - 2K(x)e^u \right) - 4 \int_{\partial\Sigma} he^{u/2}.
\]

By minimizing the Euler-Lagrange energy and via min-max methods, the authors obtain new existence results. One of the main tools in the approach is a blow-up analysis of the solutions.

The main goal is to give a precise formula for the topological degree of solutions to (1) and related versions (without geometrical mass constraints) in spirit of [Chen-Lin, CPAM, 2003]. In order to do it we need:

1. To provide a careful blow-up analysis, namely to study sequence of solutions such that $\sup u_n \to +\infty$ and to control the error terms respect to the profile in a sharp way.

2. To deduce necessary conditions for blowing-up sequences on $K$ and $h$. The interaction between these functions can one think that the condition might be non-local, see [Chang-Liu, Math Z, 1996].

3. To apply the previous results to obtain solvability if $\Sigma = \mathbb{D}$. The main difficulties are due to the non-compact effect of the group of conformal transformations of the disk, as in the Nirenberg problem [Chang-Yang, Acta Math, 1987].

Since there exists a previous and fruitful collaboration with Prof. Andrea Malchiodi and a current one with Aleks Jevnikar, I believe that a research stay in Scuola Normale Superiore di Pisa can be a great opportunity to learn, obtain new results and consolidate my career.

I would be pleased to explain more details about this project in a future meeting. Honestly, I believe I can contribute to expand the research profile of this fascinating program.

Rafael López Soriano