

PROPOSED RESEARCH PROJECT DURING THE VISITING PERIOD

Our intention is to study a particular Liouville type problem in a compact surface Σ with boundary equipped with a certain metric g . Consider the following boundary value problem:

$$\begin{cases} -\Delta_g v + 2\tilde{K} = 2Ke^v & \text{in } \Sigma, \\ \frac{\partial v}{\partial n} + 2\tilde{h} = 2he^{v/2} & \text{on } \partial\Sigma. \end{cases} \quad (1)$$

This kind of equations has a special interest due to its geometric and physical meaning. Indeed, the problem allows us to prescribe Gaussian curvature in Σ and geodesic curvature on $\partial\Sigma$.

Recently, some progresses have been made in [LS-Malchiodi-Ruiz, 2018] considering the following energy functional $I : H^1(\Sigma) \rightarrow \mathbb{R}$,

$$I(u) = \int_{\Sigma} \left(\frac{1}{2} |\nabla u|^2 + 2\tilde{K}u - 2K(x)e^u \right) - 4 \int_{\partial\Sigma} he^{u/2}. \quad (2)$$

By minimizing the Euler-Lagrange energy and via min-max methods, the authors obtain new existence results. One of the main tools in the approach is a blow-up analysis of the solutions.

The main goal is to give a precise formula for the topological degree of solutions to (1) and related versions (without geometrical mass constraints) in spirit of [Chen-Lin,CPAM,2003]. In order to do it we need:

1. To provide a careful blow-up analysis, namely to study sequence of solutions such that $\sup u_n \rightarrow +\infty$ and to control the error terms respect to the profile in a sharp way.
2. To deduce necessary conditions for blowing-up sequences on K and h . The interaction between these functions can one think that the condition might be non-local, see [Chang-Liu,Math Z, 1996].
3. To apply the previous results to obtain solvability if $\Sigma = \mathbb{D}^2$. The main difficulties are due to the non-compact effect of the group of conformal transformations of the disk, as in the Nirenberg problem [Chang-Yang,Acta Math,1987].

Since there exists a previous and fruitful collaboration with Prof. Andrea Malchiodi and a current one with Aleks Jevnikar, I believe that a research stay in Scuola Normale Superiore di Pisa can be a great opportunity to learn, obtain new results and consolidate my career.

I would be pleased to explain more details about this project in a future meeting. Honestly, I believe I can contribute to expand the research profile of this fascinating program.

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