ALTERING THE REMOTE PAST

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By "remote past" I mean "the past, many miles away"; or just "long ago." Either way, the possibility of altering it is also remote, but perhaps worth considering nonetheless.

Effect of waiting: Two trains collided at noon, 1 January 2000, in Tokyo—if we make a certain measurement today in Pittsburgh at 5 o'clock. But if we wait and make the same measurement today at 6 o'clock (*instead*), the same Japanese trains passed each other without incident at noon, 1/1/2000.

Angles & times

- “Bell’s observable” is function of four quantities, two for each side
- usually angles (directions)
- times instead!
- violation of inequality (assuming *realism*) indicates violation of appropriate *parameter independence*
- **angles**: value possessed by a system is modified by physical rotation of distant apparatus
- **times**: value possessed by a system at time t depends on whether one *waits* (far away), *i.e.* on whether a distant measurement is made at t' or at t''

Bell's inequality

- many pairs

$$\left(\mathfrak{O}^1(1),\mathfrak{O}^2(1)\right),\ldots,\left(\mathfrak{O}^1(N),\mathfrak{O}^2(N)\right)$$

  of objects

- each $\mathfrak{O}^s(k)$ assumed to possess two-valued property

$$\underline{\sigma}^s_{m,n}(k) = \pm 1$$

  characterized by parameters m, n $(s = 1, 2)$

- once $s \in \{1,2\}$ and $k \in \{1,\ldots,N\}$ are fixed, $\underline{\sigma}^s_{m,n}(k)$ is function of arguments m and n

- modulus of

$$\begin{aligned}\underline{B}(k) = {} & \underline{\sigma}^1_{a,b}(k)\underline{\sigma}^2_{b,a}(k) - \underline{\sigma}^1_{a,b'}(k)\underline{\sigma}^2_{b',a}(k) \\ & + \underline{\sigma}^1_{a',b}(k)\underline{\sigma}^2_{b,a'}(k) + \underline{\sigma}^1_{a',b'}(k)\underline{\sigma}^2_{b',a'}(k)\end{aligned}$$

  can reach 4

- *parameter independence*: value of $\underline{\sigma}^{s}_{m,n}(k)$ depends only on first parameter m (and not on second)

$\Rightarrow$ can drop second subscripts:

$$\underline{B}(k) = \underline{\sigma}^{1}_{a}(k)\{\underline{\sigma}^{2}_{b}(k) - \underline{\sigma}^{2}_{b'}(k)\} + \underline{\sigma}^{1}_{a'}(k)\{\underline{\sigma}^{2}_{b}(k) + \underline{\sigma}^{2}_{b'}(k)\}$$

$\Rightarrow$ *halves* bound on modulus of $\underline{B}(k)$

$\Rightarrow$ Bell inequality: $-2 \le \underline{B} \le 2$, where

$$\underline{B} = \frac{1}{N}\sum_{k=1}^{N}\underline{B}(k)$$

- can also write

$$\underline{B} = \underline{P}(a,b) - \underline{P}(a,b') + \underline{P}(a',b) + \underline{P}(a',b')$$

where

$$\underline{P}(m,n) = \frac{1}{N}\sum_{k=1}^{N}\underline{\sigma}^{1}_{m}(k)\underline{\sigma}^{2}_{n}(k)$$

Quantum mechanics

- *generalized Pauli operators*: unitary self-adjoint on $\mathbb{C}^2$, vanishing trace
- characterized by two angles φ, θ
- one is enough, $\theta = \theta_0$ can be left fixed:

$$\sigma_\varphi = \sigma_{(\varphi,\theta_0)} = |\varphi_+\rangle\langle\varphi_+| - |\varphi_-\rangle\langle\varphi_-|$$

  (the $|\varphi_\pm\rangle$ are orthonormal)

- $\mathbb{C}^2 \otimes \mathbb{C}^2$: the average $\langle \Sigma | B | \Sigma \rangle$ for

$$|\Sigma\rangle = \frac{1}{\sqrt{2}}\Big(|\varphi_+^1 \varphi_-^2\rangle - |\varphi_-^1 \varphi_+^2\rangle\Big)$$

  reaches maximum $2\sqrt{2}$ when operators in *Bell's observable*

$$B = \sigma_\alpha^1 \otimes \sigma_\beta^2 - \sigma_{\alpha'}^1 \otimes \sigma_\beta^2 + \sigma_\alpha^1 \otimes \sigma_{\beta'}^2 + \sigma_{\alpha'}^1 \otimes \sigma_{\beta'}^2$$

  are evenly spaced at intervals of $\dfrac{\pi}{2}$

- for instance:

$$\alpha = \beta - \frac{\pi}{4} = \alpha' - \frac{2\pi}{4} = \beta' - \frac{3\pi}{4}$$

- letting α vanish we can write

$$\begin{aligned} B = &\sigma^1_0 \otimes \sigma^2_{\pi/4} - \sigma^1_{\pi/2} \otimes \sigma^2_{\pi/4} \\ &+ \sigma^1_0 \otimes \sigma^2_{3\pi/4} + \sigma^1_{\pi/2} \otimes \sigma^2_{3\pi/4} \end{aligned}$$

- for any pair α, α' of angles

$$\begin{aligned} \exists\, U_{\Delta\alpha} &= e^{i\Delta\alpha} |+\rangle\langle+| + |-\rangle\langle-| \\ &= e^{i\Delta\alpha |+\rangle\langle+|} \end{aligned}$$

  such that $\sigma_{\alpha'} = U_{\Delta\alpha}\, \sigma_\alpha U_{-\Delta\alpha}$

  ($\Delta\alpha = \alpha' - \alpha$, the $|\pm\rangle$ are orthonormal)

- assume $|\pm\rangle$ are eigenvectors of *maximal* time-independent Hamiltonian

$$H = E_+ |+\rangle\langle +| + E_- |-\rangle\langle -|$$

$\Rightarrow$ for any pair α, α' of angles

$$\exists\, e^{iHt} = e^{iE_+ t} |+\rangle\langle +| + e^{iE_- t} |-\rangle\langle -|$$

such that $\sigma_{\alpha'} = e^{iHt} \sigma_\alpha e^{-iHt}$

$\Rightarrow$ generalized Pauli operators parametrized by times

$\Rightarrow$ times $\leftrightarrow$ angles

- we can write $\sigma_{t'} = e^{iH(t'-t)} \sigma_t e^{iH(t-t')}$

  for any pair of times t, t'

- $\mathbb{C}^2 \otimes \mathbb{C}^2$:

  $\tilde{B} = \sigma^1_t \otimes \sigma^2_u - \sigma^1_t \otimes \sigma^2_{u'} + \sigma^1_{t'} \otimes \sigma^2_u + \sigma^1_{t'} \otimes \sigma^2_{u'}$

  (u, u' are also times)

- also

$$\langle \varSigma | \tilde{B} | \varSigma \rangle = P(t,u) - P(t,u') + P(t',u) + P(t',u')$$

where

$$P(m,n) = \langle \varSigma(m,n) | \sigma_0^1 \otimes \sigma_0^2 | \varSigma(m,n) \rangle = \cos\{\Delta E(n-m)\}$$

$$|\varSigma(m,n)\rangle = (e^{iHm} \otimes e^{iHn}) |\varSigma\rangle$$

$$(\Delta E = E_- - E_+)$$

- $\langle \varSigma | \tilde{B} | \varSigma \rangle$ reaches maximum $2\sqrt{2}$ at

$$t = \frac{t_0}{\Delta E} \qquad t' = \frac{1}{\Delta E}\left(t_0 + \frac{\pi}{2}\right)$$

$$u = \frac{1}{\Delta E}\left(t_0 + \frac{\pi}{4}\right) \qquad u' = \frac{1}{\Delta E}\left(t_0 + \frac{3\pi}{4}\right)$$

(t_0 is arbitrary initial time)

- assume pairs $\left(\mathfrak{O}^1(k), \mathfrak{O}^2(k)\right)$ are accurately described by $|\Sigma(m,n)\rangle$
- and that measurement of σ_m^s faithfully reveals corresponding properties $\underline{\sigma}_m^s(k)$ $(k = 1, \ldots, N)$

$\Rightarrow$ parameter independence violated

Set-theoretical predicate

A *Bell scheme* is a system

$$\Omega = \left(\Xi, \mathfrak{O}^s(k), \underline{\sigma}_n^s(k), \underline{B}; |\Sigma\rangle, \sigma_n^m, B\right)$$

'satisfying' the following axioms:

1. $\Xi = \left\{\left(\mathfrak{O}^1(1), \mathfrak{O}^2(1)\right), \ldots, \left(\mathfrak{O}^1(N), \mathfrak{O}^2(N)\right)\right\}$ is a large ensemble of pairs of objects.
2. Object $\mathfrak{O}^s(k)$ has intrinsic properties $\underline{\sigma}_n^s(k) = \pm 1$ ($s = 1, 2$; $k = 1, \ldots, N$; all n).
3. $$\underline{B} = \frac{1}{N}\sum_{k=1}^{N}[\underline{\sigma}_\alpha^1(k)\underline{\sigma}_\beta^2(k) - \underline{\sigma}_\alpha^1(k)\underline{\sigma}_{\beta'}^2(k) + \underline{\sigma}_{\alpha'}^1(k)\underline{\sigma}_\beta^2(k) + \underline{\sigma}_{\alpha'}^1(k)\underline{\sigma}_{\beta'}^2(k)].$$
4. The value $\underline{\sigma}_n^s(k)$ does not (as the notation suggests) depend on the subscript of the adjacent factor.

5. $\varXi$ is described by the quantum state vector

$$|\varSigma\rangle = \frac{1}{\sqrt{2}}\left(|\varphi_+^1 \varphi_-^2\rangle - |\varphi_-^1 \varphi_+^2\rangle\right) \in \mathbb{C}^{2(1)} \otimes \mathbb{C}^{2(2)}.$$

6. The operator $\sigma_n^s : \mathbb{C}^{2(s)} \to \mathbb{C}^{2(s)}$ is self-adjoint, unitary and zero trace (one degree of freedom has been dropped, leaving the single parameter n).

7. Measurement represented by σ_n^s faithfully reveals property $\underline{\sigma}_n^s(k)$ $(s = 1,2;\ k = 1,\ldots,N;$ all $n)$.

8. $B = \sigma_\alpha^1 \otimes \sigma_\beta^2 - \sigma_\alpha^1 \otimes \sigma_{\beta'}^2 + \sigma_{\alpha'}^1 \otimes \sigma_\beta^2 + \sigma_{\alpha'}^1 \otimes \sigma_{\beta'}^2.$

- strictly speaking the axioms have no ('classical') models, since they are inconsistent:

$$5,6,8 \Rightarrow \max\left(\langle \Sigma \mid B \mid \Sigma \rangle\right) = 2\sqrt{2}$$

$$2,4\ (\&1,3) \Rightarrow -2 \leq \underline{B} \leq 2$$

$$3,7,8\ (\&1,2,6) \Rightarrow \langle \Psi \mid B \mid \Psi \rangle = \underline{B}\ \forall \mid \Psi \rangle$$

$$2,5,7\ (\&1,3,6,8) \Rightarrow \max(\underline{B}) = 2\sqrt{2}$$

$$2,4,7\ (\&1,3,6,8) \Rightarrow -2 \leq \langle \Psi \mid B \mid \Psi \rangle \leq 2\ \forall \mid \Psi \rangle$$

- something is wrong, who knows what
- axioms 1,3,6,8 are definitions
- axioms 2 and 7 go together, both plausible
- axiom 4 is the very one at issue
- axiom 5 expresses validity of QM
- will make no attempt to resolve the inconsistency, which conveniently expresses the tension at issue
- besides, nature itself may be inconsistent, in a way that is somehow represented by the inconsistency of the axioms

Altering the remote past

- for modulus of

$$\underline{B} = \frac{1}{N}\sum_{k=1}^{N}\underline{B}(k)$$

  to exceed 2, modulus of

$$\underline{B}(k) = \underline{\sigma}^1_{t,u}(k)\underline{\sigma}^2_{u,t}(k) - \underline{\sigma}^1_{t,u'}(k)\underline{\sigma}^2_{u',t}(k) + \underline{\sigma}^1_{t',u}(k)\underline{\sigma}^2_{u,t'}(k) + \underline{\sigma}^1_{t',u'}(k)\underline{\sigma}^2_{u',t'}(k)$$

  must also exceed 2 for at least one k

- if we suppose that $\underline{B}(k) \neq 2$ for $k = k_0$, there must be at least one h such that

$$\underline{\sigma}^s_{h,j}(k_0) \neq \underline{\sigma}^s_{h,j'}(k_0)$$

- suppose

$$\underline{\sigma}^1_{t',u}(k_0) \neq \underline{\sigma}^1_{t',u'}(k_0)$$

  and that the last two terms of $\underline{B}(k_0)$ are $(-1)\cdot\underline{\sigma}^2_{u}(k_0)$ and $(+1)\cdot\underline{\sigma}^2_{u'}(k_0)$

- at first sight this seems impossible

- perhaps it is, in which case either quantum mechanics is wrong, or underlined expressions like $\underline{B}$ or $\underline{\sigma}^{s}_{m,n}(k)$ make no sense in the first place

- but here we are assuming that quantum mechanics works, and that underlined expressions do make sense, and exploring the implications

- so we must wonder how it is that

  $$\underline{\sigma}^{1}_{t',u}(k_0) = -1 \neq +1 = \underline{\sigma}^{1}_{t',u'}(k_0)$$

  i.e. that

  $$\underline{\sigma}^{1}_{t'}(k_0) = -1$$

  when (first) subscript of adjacent factor is u whereas

  $$\underline{\sigma}^{1}_{t'}(k_0) = +1$$

  when subscript of neighbouring factor is u'

- surely it makes no sense to say that

$$\underline{\sigma}_{t'}^{1}(k_0) = -1$$

when $\underline{\sigma}_{t'}^{1}(k_0)$ is *written down* or *considered* alongside $\underline{\sigma}_{u}^{2}(k_0)$, but

$$\underline{\sigma}_{t'}^{1}(k_0) = +1$$

when $\underline{\sigma}_{t'}^{1}(k_0)$ appears beside $\underline{\sigma}_{u'}^{2}(k_0)$

- the dependence must have more substance to it than that, it must be more than an abstract 'notational' association

- the apparatus may do no more than faithfully reveal a value that was there anyway, but surely the mere *consideration* of $\underline{\sigma}_{u'}^{2}(k_0)$ rather than $\underline{\sigma}_{u}^{2}(k_0)$, the fact that we express more of an interest in the former than in the latter, cannot change the value of $\underline{\sigma}_{t}^{1}(k_0)$

- so *measurement* would appear to matter: if the dependence of $\underline{\sigma}^1_{t',n}(k_0)$ on the second index n is to make any sense at all, the last two terms of $\underline{B}(k_0)$ must refer to different experimental situations

- the product

$$\underline{\sigma}^1_{t'}(k_0)\underline{\sigma}^2_u(k_0)$$

  must refer to the two measurements characterized by t' and u, the fourth term to the measurements characterized by t' and u'

- the choice of measuring $\sigma^2_{u'}$ rather than σ^2_u, and hence of revealing $\underline{\sigma}^2_{u'}(k_0)$ rather than $\underline{\sigma}^2_u(k_0)$, corresponds to a physical circumstance; the effect must be somehow due to that circumstance

- where the parameter is a direction representing the orientation of an apparatus, the circumstance is a rotation, and that's surprising enough

- but now that the parameter is a time, the *very same* quantity is measured at times u and u'
- the apparatus remains unchanged; it does exactly the same thing, only later

Dilemma

We have something of a dilemma, concerning the role of measurement. Since

$$\underline{\sigma}^{1}_{t',u}(k_0) \neq \underline{\sigma}^{1}_{t',u'}(k_0),$$

the value $\underline{\sigma}^{1}_{t',n}(k_0)$ appears to depend on the second index n, which refers to the other object of the pair. It identifies a particular property of the other object, namely the 'time-n property.' We have assumed that measurement does no more than faithfully reveal the property that was there anyway, and in no sense creates the property. But how can the time of a measurement affect a distant outcome? Where the parameter n represents an angle, the effect would generally be attributed to the *physical rotation of the measuring apparatus on the other side*. But here, with times rather than angles, there seems to be no physical change worth speaking of; the experimenter just waits, and does exactly the same thing sooner rather than later. Besides, what if

$$\underline{\sigma}^{1}_{t',u}(k_0) \neq \underline{\sigma}^{1}_{t',u'}(k_0)$$

with $t' < u, u'$? Quite apart from any *change* due to the choice of t' or t'', does the first object have *any* value before the measurement on the other side is made? If the value $\underline{\sigma}^{1}_{t',n}(k_0)$ of the first object at time t' does indeed depend on the time n at which *measurement* is performed on the other object, what value should $\underline{\sigma}^{1}_{t',n}(k_0)$ be given before that second measurement? What if no measurement is made on the second object? Does the first object have any value in that case?

It is far from clear how how *waiting* can change a value possessed by an object that could be spatially and temporally remote. But let us assume it can, and consider the implications. To begin with, the properties $\underline{\sigma}^{s}_{m,n}(k) = \pm 1$ could be linked to larger circumstances to amplify the effects in question: the trains.

So violation of parameter independence suggests that a property $\underline{\sigma}^{s}_{t}(k)$ of an object—and hence the fate of a train—can be changed by making a distant measurement at a time t' rather than at another time t''. Nothing is said, beyond $t' \neq t''$, about the order of t, t' and t''. Nature and common sense would appear to favour

$$t > t' > t'' \quad \text{or} \quad t > t'' > t'$$

over the other four possible orderings, for it seems easier to change the future than the past. But should

$$t' > t > t'', \quad t' > t'' > t, \quad t'' > t > t', \quad t'' > t' > t$$

really be ruled out? The whole formalism, which makes no distinction between past and future, is so impartial toward all six orderings that one has to wonder. When the parameter is an angle, does it matter whether the apparatus is turned clockwise or anti-clockwise?

Where $t' > t > t''$, for instance, nothing is done at t''; then the value of $\underline{\sigma}_t^s(k)$ changes; then the measurement that was not made at t'' is made at t' instead. What is it that changes$\underline{\sigma}_t^s(k)$? Is it that nothing was done at t'', in the past? Or is it that a measurement *will be* made at t', in the future? Is it both? The distant change could depend on the difference $t' - t''$, in which case the influence would straddle the present and belong partly to the past and partly to the future.

Suppose $t'' > t' > t$. Should we just speak of a *correlation* between the outcome at t and the choice of measuring at t' or t'', or can we really identify the choice of measuring at t' or t'' as the *cause*, the outcome at t as the *effect*? Maybe the outcome at t, which occurs first, influences the choice. The choice of measuring at t' or t'' could, in principle, depend on the outcome at t; but it can also be made independent. One can, for instance, appeal to the free will of the experimenter, who can decide whether to measure at t' or t'' regardless of what happened at t; or one can rely on a random process, like a random number generator, to decide between t' and t''. Surely a random number generator can be built whose output does not depend on the outcome of a measurement performed years before.

The effect at issue here, if indeed present, seems difficult to control, to exploit in any useful way. Suppose $\underline{\sigma}_t^1(1) = +1$. The measurement on the second object is then made at, say, $t' > t$. We then get, say, $\underline{\sigma}_t^1(2) = -1$, and $\underline{\sigma}_t^1(3) = +1$. But what then? All we have established is that a subsequent measurement on the second object may change $\underline{\sigma}_t^1(3) = +1$ to $\underline{\sigma}_t^1(3) = -1$. We know nothing, however, about which times will produce the change, and which will not. For instance, it could be that $\underline{\sigma}^1_{t,t'}(3) = -1$.

Relativity

An appeal to relativity hardly clarifies matters. So far we have spoken of the 'absolute' times t, t' and t''. In a relativistic treatment $t = t_r(\eta^{\pm})$, $t' = t_r(\eta')$ and $t'' = t_r(\eta'')$ can be considered the times of the corresponding events $\eta^{\pm}, \eta'$ and η'' with respect to some inertial system r, where $t = t_r(\eta^{\pm})$ means that *the value* ± 1 *is possessed at a spacetime point whose time coordinate is* t *with respect to* r; and η', η'' are the measurements made on the other object of the same pair. Although η' and η'', which concern the same system, should not be spacelike separated, the relationship between $\eta^{\pm}$ and η', η'' is arbitrary. If η', η'' are in the past light cone of $\eta^{\pm}$, relativity would allow either one, or even both, to influence $\eta^{\pm}$, for the times of η', η'' would precede that of $\eta^{\pm}$ with respect to all inertial systems. But events η', η'' could also lie in the absolute future of $\eta^{\pm}$; the choice of measuring at t' and not at t'' could then turn $\eta^{\pm}$ into the event $\eta^{\mp}$ occupying the same spacetime point in the absolute past of η', η''. And what if η', η'' are both spacelike separated from η? Then Lorentz transformations could change the order from

$$t_{r_1}(\eta^{\pm}) > t_{r_1}(\eta') > t_{r_1}(\eta'')$$

to

$$t_{r_2}(\eta') > t_{r_2}(\eta^{\pm}) > t_{r_2}(\eta'')$$

to

$$t_{r_3}(\eta') > t_{r_3}(\eta'') > t_{r_3}(\eta^{\pm}),$$

or from

$$t_{r_1'}(\eta^{\pm}) > t_{r_1'}(\eta'') > t_{r_1'}(\eta')$$

to

$$t_{r_2'}(\eta'') > t_{r_2'}(\eta^{\pm}) > t_{r_2'}(\eta')$$

to

$$t_{r_3'}(\eta'') > t_{r_3'}(\eta') > t_{r_3'}(\eta^{\pm}).$$

So can an influence going from the past to the future be turned into an influence going from the future to the past, or from the past and future to the present, by a Lorentz transformation? Maybe it makes no sense to speak of *past* and *future* in cases disallowed by relativity theory in the first place.

Conclusions

The emphasis has been not just on altering the past, but on doing so by *waiting*. Of course the possibility of changing the past is not to be taken too seriously; here it is only viewed as a consequence of quantum mechanics together with an appropriate kind of realism. The whole matter can be treated as a *reductio* argument against realism or quantum mechanics; or, if all the assumptions are to be taken seriously, I suppose one could actually wonder about altering the past, and start thinking about realizations. The possibility is of course highly paradoxical, and difficult to reconcile with most received ideas about causality.

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